7.1 – Solving Linear Systems by Graphing

-KEY-

Learning Target: to solve linear systems by graphing

Toolkit:
- graphing lines
- rewriting equations into $y = mx + b$ form
- substitution

Linear System - Many problems in mathematics are defined with two equations called a **system of linear equations.**

Solving a System – to SOLVE a linear system, find the coordinates where the two lines intersect (the point where the two lines cross). You will have an x and a y value!

Steps for solving systems graphically:

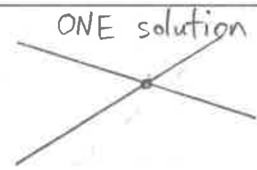
1. Change each equation to a form that is easy to graph ($y = mx + b$ **OR** $Ax + By = C$)
2. Graph each line on the SAME GRID
3. Identify the point of intersection of the two lines.

** The **solution** of the system is the ordered pair (x, y) of the point of intersection.

4. Check the solution by substituting the ordered pair into each equation of the original system.

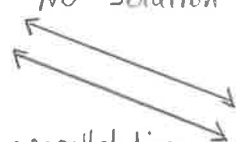
What are the three possibilities for **number of intersections** when two lines are graphed?

1) ONE solution



- different slopes

2) NO Solution



- parallel lines
  - same slopes
  - different y-intercepts

3) Infinite Solutions

- same line!
  - same slopes
  - same y-ints

Ex 1) Solve the system graphically and check the solution

1) $x + y = 5$
2) $2x + 3y = 12$

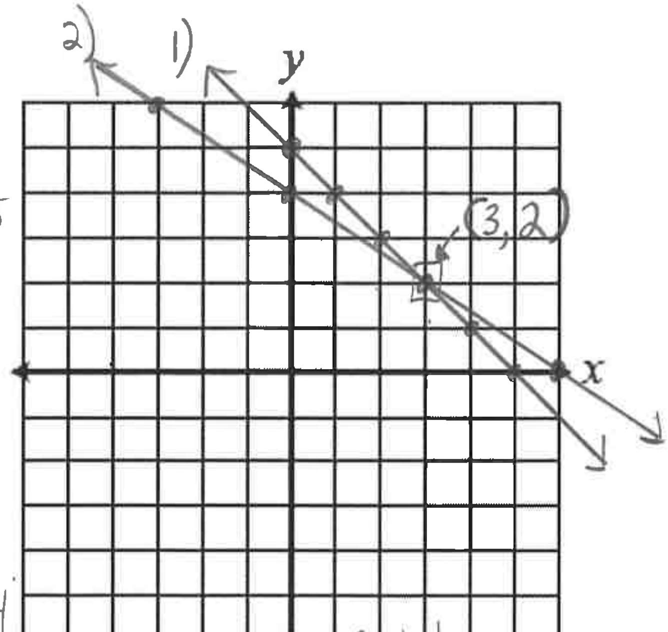
1) $x + y = 5$
$\quad -x \qquad -x$

$y = -x + 5 \rightarrow$ y-int $= +5$
$(y = mx + b)$ slope $= \frac{-1}{1}$

2) $2x + 3y = 12$
$\quad -2x \qquad -2x$

$\frac{3y}{3} = \frac{-2x}{3} + \frac{12}{3}$

$y = -\frac{2}{3}x + 4 \rightarrow$ y-int $= 4$
slope $= \frac{-2}{3}$



$\boxed{\text{solution is } (3,2)}$

check

1) $x + y = 5$
$(3) + (2) = 5$
$5 = 5$ ✓

2) $2x + 3y = 12$
$2(3) + 3(2) = 12$
$6 + 6 = 12$
$12 = 12$ ✓

What if you just need to check?

Ex 2) Is $(2, -1)$ a solution to the following system? 1) $3x + 5y = 1$
2) $2x - 2y = 5$

x y

1) check $3x+5y=1$
$3(2)+5(-1)=1$
$6-5=1$
$1=1$ ✓

2) $2x-2y=5$
$2(2)-2(-1)=5$
$4+2=5$
$6\neq 5$ ✗

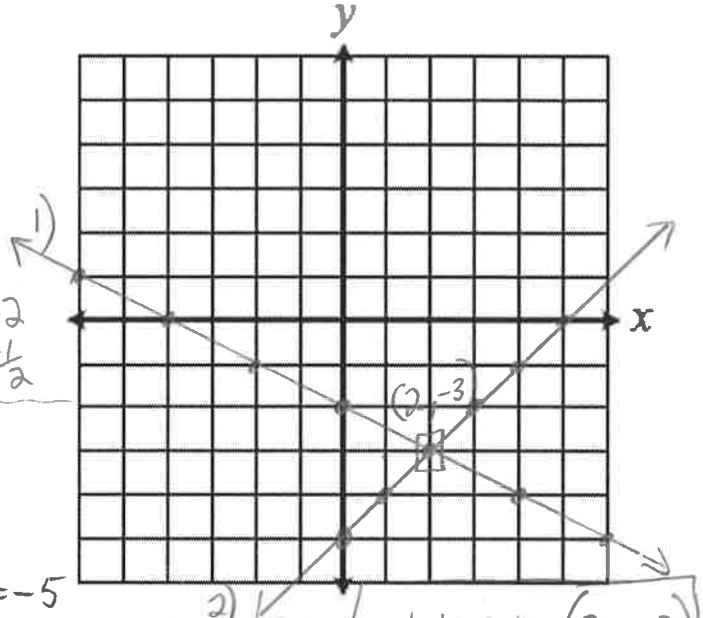
(2,-1) does not satisfy both equations, ∴ is not a solution!

Ex 3) Solve the system by graphing

1) $x + 2y = -4$
2) $x - y = 5$

1) $x+2y=-4$
$-x \quad -x$
$\frac{2y}{2} = \frac{-x}{2} - \frac{4}{2}$
$y = -\frac{1}{2}x - 2 \rightarrow$ y-int $= -2$
slope $= -\frac{1}{2}$

2) $x - y = 5$
$-x \quad -x$
$\frac{-y}{-1} = \frac{-x}{-1} + \frac{5}{-1}$
$y = x - 5 \rightarrow$ y-int $= -5$
slope $= \frac{1}{1}$



solution is $(2,-3)$

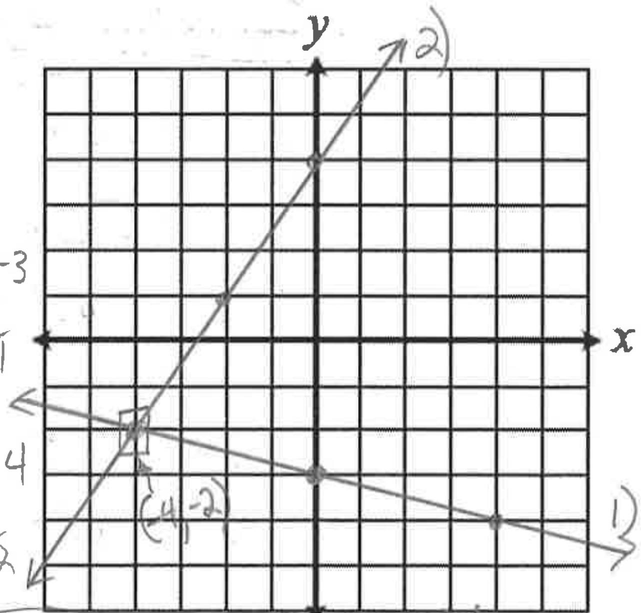
"f(x)", "g(x)" same as "y"!

Ex 4) Solve the system by graphing

1) $\underbrace{f(x)}_{y} = -\frac{1}{4}x - 3$
2) $\underbrace{g(x)}_{y} = \frac{3}{2}x + 4$

1) $y = -\frac{1}{4}x - 3 \rightarrow$ y-int $= -3$
slope $= \frac{-1}{4}$

2) $y = \frac{3}{2}x + 4 \rightarrow$ y-int $= 4$
slope $= \frac{3}{2}$



solution is $(-4,-2)$

7.2 – Solving Linear Systems by Addition (Elimination)

Learning Target: to solve a system of linear equations algebraically, by adding the equations together

Toolkit:

- substitution
- rearranging equations
- coefficient: the constant (#) in front of variable

ex. $3x \rightarrow$ coefficient is 3

Solving a linear system by graphing is limited by the accuracy of the graph. Also, when the intersection point is not coordinates that are exact integers, it's difficult to determine the exact coordinates from the graph.

Solving a Linear System by the Addition Method (also known as the Elimination Method)

1) Write the equations in STANDARD form ($Ax + By = C$)
   - You MAY have a negative "Ax" term here
   - This step *may not be necessary*
2) Multiply the terms of one equation, or both equations, by a constant *(if necessary)* so that the coefficients of x *or* y are different ONLY IN THEIR SIGN
3) ADD the equations to eliminate either x *or* y, and SOLVE the resulting equation
4) Substitute the value obtained in step 3 into either of the original equations, and solve for the remaining variable.
5) Write the solution to the system as (x, y)
6) Check that the solution satisfies each of the original equations

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Ex 1) Solve the system by the Addition Method

write in $Ax+By=C$

$-5y \ -9 \quad -5y \ -9$

1) $3x + 9 = 5y$
2) $4x + 5y = 23$

$-5y$ and $5y$ only different in their sign!

$$\begin{aligned} 1)\ & 3x - 5y = -9 \\ +2)\ & 4x + 5y = 23 \\ \hline & 7x + 0y = 14 \\ & 7x = 14 \\ & \tfrac{7x}{7} = \tfrac{14}{7} \\ & \boxed{x = 2} \end{aligned}$$

choose $4x + 5y = 23$ ... to solve for y!

$$\begin{aligned} 4x + 5y &= 23 \\ 4(2) + 5y &= 23 \\ 8 + 5y &= 23 \\ -8 \quad & \quad -8 \\ 5y &= 15 \\ \tfrac{5y}{5} &= \tfrac{15}{5} \\ \boxed{y = 3} \end{aligned}$$

solution is $\boxed{(2, 3)}$

check

1) $3x+9=5y$	2) $4x+5y=23$
$3(2)+9=5(3)$	$4(2)+5(3)=23$
$6+9=15$	$8+15=23$
$15=15$ ✓	$23=23$ ✓

Ex 2) Solve by the Addition Method:

$+4x \quad +4x$

1) $3y = -4x + 5$
2) $4x - 7y = 15$

$$1)\ 4x + 3y = 5$$
$$2)\ (4x - 7y = 15) \times -1$$

now, $4x$ and $-4x$ differ only in their sign!

$$\begin{aligned} 1)\ & 4x + 3y = 5 \\ +2)\ & -4x + 7y = -15 \\ \hline & 0x + 10y = -10 \\ & 10y = -10 \\ & \tfrac{10y}{10} = \tfrac{-10}{10} \\ & \boxed{y = -1} \end{aligned}$$

choose $4x - 7y = 15$

$$\begin{aligned} 4x - 7y &= 15 \\ 4x - 7(-1) &= 15 \\ 4x + 7 &= 15 \\ -7 \quad & \quad -7 \\ 4x &= 8 \\ \tfrac{4x}{4} &= \tfrac{8}{4} \\ \boxed{x = 2} \end{aligned}$$

solution $\boxed{(2, -1)}$

make the $\underline{y}$-terms $10y$ and $-10y$

Ex 3) Solve by the Addition Method:

1) $2x + 5y = 11$
2) $-2y = -3x + 7$
$\quad +3x \qquad +3x$

1) $(2x + 5y = 11) \times 2$
2) $(3x - 2y = 7) \times 5$

$$
\begin{array}{r}
\text{1) } 4x + 10y = 22 \\
+\ 15x - 10y = 35 \\
\hline
19x + 0y = 57
\end{array}
$$

$\frac{19x}{19} = \frac{57}{19}$

$\boxed{x = 3}$

choose $2x + 5y = 11$

$2x + 5y = 11$

$2(3) + 5y = 11$

$6 + 5y = 11$
$-6 \qquad -6$

$\frac{5y}{5} = \frac{5}{5}$

$\boxed{y = 1}$

$\boxed{\text{solution } (3, 1)}$

Ex 4) Solve by the Addition Method: clear fractions by multiplying all terms by the L.C.D.!

1) $\frac{3}{4}x - y = 2$ (×4 ×4 ×4) → 1) $3x - 4y = 8$

2) $\frac{1}{8}x + \frac{1}{4}y = 2$ (×8 ×8 ×8) → 2) $(x + 2y = 16) \times 2$ → to get $-\underline{4}y$ and $\underline{4}y$

$$
\begin{array}{r}
\text{1) } 3x - 4y = 8 \\
+\ \text{2) } 2x + 4y = 32 \\
\hline
5x + 0y = 40
\end{array}
$$

$\frac{5x}{5} = \frac{40}{5}$

$\boxed{x = 8}$ sub into $\underline{any}$ eqn!

choose $3x - 4y = 8$

$3(8) - 4y = 8$

$24 - 4y = 8$
$-24 \qquad -24$

$\frac{-4y}{-4} = \frac{-16}{-4}$

$\boxed{y = 4}$

$\boxed{\text{solution } (8, 4)}$

7.3 – Solving Linear Systems by Substitution

Learning Target: To use the substitution of one variable to solve a linear system algebraically

Toolkit:
- Rearranging equations
- Substituting values into equations
- A **solution to a linear system** is an (x, y) ordered pair where two lines cross

Another way to solve linear systems algebraically is called **Substitution**

Solving a Linear System by the Substitution Method:

1) Solve one equation for one of its variables in terms of the other variable; this becomes equation 3
2) Substitute the equation from step 1 into the other equation, and then SOLVE that equation.
3) Take the value solved for in step 2, and substitute the value into equation 3 to find the other value
4) Write the solution as an (x, y) ordered pair
5) Check that the solution satisfies both equations

Ex 1) Solve by Substitution and check

1) $3x + y = 3$ ← easiest to solve for y in eqn. 1

$-3x \quad -3x$

2) $7x - 2y = 20$

3) $y = -3x + 3$

2) $7x - 2y = 20$

now solve for x!

$7x - 2(-3x + 3) = 20$

$7x + 6x - 6 = 20$

$+6 \quad +6$

$13x = 26$

$\frac{13x}{13} = \frac{26}{13}$

$\boxed{x = 2}$

3) $y = -3x + 3$

$y = -3(2) + 3$

$y = -6 + 3$

$\boxed{y = -3}$

$\boxed{\text{solution } (2, -3)}$

check:

① $3x + y = 3$

$3(2) + (-3) = 3$

$6 - 3 = 3$

$3 = 3$ ✓

② $7x - 2y = 20$

$7(2) - 2(-3) = 20$

$14 + 6 = 20$

$20 = 20$ ✓

Ex 2) Solve by Substitution

1) $y = 3x + 2$ → 3) $y = 3x + 2$

2) $y + x = -14$

$(3x + 2) + x = -14$

$3x + 2 + x = -14$

$-2 \quad -2$

$4x = -16$

$\frac{4x}{4} = \frac{-16}{4}$

$\boxed{x = -4}$

3) $y = 3x + 2$

$y = 3(-4) + 2$

$y = -12 + 2$

$\boxed{y = -10}$

$\boxed{\text{solution } (-4, -10)}$

Ex 3) Solve by Substitution

$1)\ 2x - 4y = 7$
$2)\ -x + 8y = -5$ ← easiest to solve for x in eqn 2

$\quad\quad -8y \quad -8y$

$\frac{-x}{-1} = \frac{-8y}{-1} - \frac{5}{-1}$

$3)\ x = 8y + 5$

$1)\ 2x - 4y = 7$

$2(8y+5) - 4y = 7$

$16y + 10 - 4y = 7$

$\quad\ -10 \quad\quad -10$

$\frac{12y}{12} = \frac{-3}{12}$ $\div 3$

$\boxed{y = -\frac{1}{4}}$

$3)\ x = 8y + 5$

$x = 8\left(-\frac{1}{4}\right) + 5$

$x = -\frac{8}{4} + 5$

$x = -2 + 5$

$\boxed{x = 3}$

$\boxed{\text{solution } \left(\overset{x}{3}, \overset{y}{-\frac{1}{4}}\right)}$

Ex 4) Solve by Substitution

clear fractions by multiplying each eqn by the L.C.D.!

$1)\ \left(-\frac{x}{5}\right)^{\times 15} + \left(\frac{y}{3}\right)^{\times 15} = \left(\frac{2}{15}\right)^{\times 15}$

$2)\ \left(\frac{x}{7}\right)^{\times 7} = \left(-y\right)^{\times 7}$

$1)\ -3x + 5y = 2$

$2)\ x = -7y$ ← x already by itself!

$3)\ x = -7y$

$1)\ -3x + 5y = 2$

$-3(-7y) + 5y = 2$

$21y + 5y = 2$

$\frac{26y}{26} = \frac{2}{26}$

$\boxed{y = \frac{1}{13}}$

$3)\ x = -7y$

$x = -7\left(\frac{1}{13}\right)$

$\boxed{x = -\frac{7}{13}}$

$\boxed{\text{solution } \left(\overset{x}{\frac{-7}{13}}, \overset{y}{\frac{1}{13}}\right)}$

7.4A – Problem Solving with Two Variables

Learning Target: to model situations and answer problems using a system of linear equations

Toolkit:

- Sum, greater than is +
- Difference, less than is –
- Times, product is x
- To change a % to a decimal, move decimal two places to the _________
  Ex. 6.5% = ____________

These word problems involve two unknowns. We need two equations to solve for two unknowns, so it will be your job to create the system of two equations and solve it!

STEPS for Solving Linear Systems Word Problems:

1) Define your two variables after reading the question over carefully and determining what you are being asked to solve for.. You may use x and y, but it is also good to practice working with other variables (such as t for time). Use "let: statements (Ex. Let x be the number of …)
2) Build your two equations, using both variables in both equations
3) Solve the system, using either the Addition Method or the Substitution Method
4) Write a sentence answer
5) Check your answer to make sure all conditions are satisfied

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solve by addition or substitution

Ex 1) The sum of two numbers is 53. The first number is 7 greater than the second. What are the numbers?

let x = first #
let y = second #

① $x + y = 53$
② $x = y + 7$

3) $x = y + 7$
① $x + y = 53$
$(y+7)+y = 53$
$\cancel{y} + 7 + \cancel{y} = 53$ (−7, −7)
$2y = 46$ (÷2, ÷2)
$\boxed{y = 23}$

3) $x = y + 7$
$x = (23) + 7$
$\boxed{x = 30}$

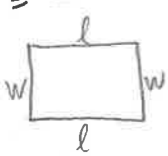
check
$23 + 30 = 53$
30 is 7 more than 23

$\boxed{\therefore \text{ The two numbers are 30 and 23}}$

Ex 2) The perimeter of a rectangle is 46cm. What are the dimensions if the length is 4cm less than twice the width?

length + width, 2w−4

let l = length , let w = width



$P = 2l + 2w$

$2l + 2w = P$
① $2l + 2w = 46$
② $l = 2w - 4$
3) $l = 2w - 4$
① $2l + 2w = 46$
$2(2w-4) + 2w = 46$
$4w - 8 + 2w = 46$ (+8, +8)
$6w = 54$ (÷6, ÷6)
$\boxed{w = 9}$

3) $l = 2w - 4$
$l = 2(9) - 4$
$l = 18 - 4$
$\boxed{l = 14}$

$\boxed{\text{The dimensions are 14cm by 9cm}}$

Ex 3) For a basketball game, 1600 total tickets were sold. Adult tickets cost \$3, and student tickets cost \$2. If the total money brought in by ticket sales was \$4000, how many of each kind of ticket were sold?

let a = # of adult tix sold
let s = # of student tix sold

solve by Addition (Elimination)

to get $-2s$ and $+2s$

$$① \; (a + s = 1600) \times -2$$

$$\underbrace{\#\text{ from } a}+\underbrace{\#\text{ from } s}=\underbrace{\text{total }\#}$$

$$② \; 3a + 2s = 4000$$

$$+ \; ① \; -2a - 2s = -3200$$

$$a + 0s = 800$$

$$\boxed{a = 800}$$

choose ①

$$a + s = 1600$$

$$(800) + s = 1600$$

$$-800 \qquad -800$$

$$\boxed{s = 800}$$

800 adult tix were sold, and 800 student tix were sold.

Ex 4) Isaac borrowed \$2100 for his college tuition. Part of it he borrowed from the government, which charged 5% annual interest. The rest he borrowed from a bank at 6.5% annual interest. If the total annual interest is \$114, how much did he borrow from each source?

$5\% = \frac{5}{100} = 0.05$

$6.5\% = \frac{6.5}{100} = 0.065$

annual = yearly

let g = amount from gov't
let b = amount from bank

$$① \; g + b = 2100$$

$$-g \qquad -g$$

$$\underbrace{\text{interest from } g}+\underbrace{\text{interest from } b}=\underbrace{\text{total interest}}$$

$$② \; 0.05g + 0.065b = 114$$

solve by substitution

$$3) \; b = 2100 - g$$

$$② \; 0.05g + 0.065b = 114$$

$$0.05g + 0.065(2100 - g) = 114$$

$$0.05g + 136.5 - 0.065g = 114$$

$$-136.5 \qquad -136.5$$

$$-0.015g = -22.5$$

$$\frac{-0.015g}{-0.015} = \frac{-22.5}{-0.015}$$

$$\boxed{g = 1500}$$

$$3) \; b = 2100 - g$$

$$b = 2100 - 1500$$

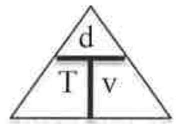
$$\boxed{b = 600}$$

Isaac borrowed \$1500 from gov't, and \$600 from the bank.

7.4B - Problem Solving with Two Variables

Learning Target: More practice with modeling situations and answering problems using a system of linear equations

Toolkit:

- $Speed = \frac{distance}{time}$ OR 

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2) Build your two equations, using both variables in both equations
3) Solve the system, using either the Addition Method or the Substitution Method
4) Write a sentence answer
5) Check your answer to make sure all conditions are satisfied

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Ex 1) The sum of two numbers is 51. One number is 3 more than twice the other number. What are the two numbers? let x = first # let y = second #

solve by substitution

① $x + y = 51$

② $x = 2y + 3$

3) $x = 2y + 3$

① $x + y = 51$

$(2y+3) + y = 51$

$2y + 3 + y = 51$

$-3 \qquad -3$

$\frac{3y}{3} = \frac{48}{3}$

$\boxed{y = 16}$

3) $x = 2y + 3$

$x = 2(16) + 3$

$x = 32 + 3$

$\boxed{x = 35}$

The numbers are 35 and 16

Ex 2) Adult tickets for the school play are \$12, and children's tickets are \$8. If a theatre holds 300 seats and the sold out performance brings in \$3280, ~~how many children, and~~ how many of each type of ticket were sold?

let a = # of adult tix sold let c = # of children's tix sold

total tix → ① $(a + c = 300) \times -8$

total # → ② $12a + 8c = 3280$

Solve by Addition (Elimination)

① $-8a - 8c = -2400$

$\frac{4a}{4} = \frac{880}{4}$

$\boxed{a = 220}$

choose ①

$a + c = 300$

$(220) + c = 300$

$-220 \qquad -220$

$\boxed{c = 80}$

220 adult tix sold, and 80 children's tix sold

against wind

Ex 3) A small airplane makes a 2400km trip in 7.5 hours, and makes the return trip in 6 hours. (with wind) If the plane travels at a constant speed, and the wind blows at a constant rate, find the airplane's airspeed, and the speed of the wind.

* ***Whenever you are doing a word problem with speed, distance, and time, it helps to set up a table like the one below:***

Let a = speed of the airplane with no wind (airspeed)

Let w = speed of the wind.

Direction	Distance (km)	Speed (velocity) km/hr	Time (h)	Equations $v = \frac{d}{t}$!
With the wind	2400	$a + w$	6	① $a + w = \frac{2400}{6}$
Against the wind	2400	$a - w$	7.5	② $a - w = \frac{2400}{7.5}$

① $a + w = \frac{2400}{6}$ → ① $a + w = 400$

② $a - w = \frac{2400}{7.5}$ $+$② $a - w = 320$

Addition (Elimination)

$$2a = 720$$

$$\frac{2a}{2} = \frac{720}{2}$$

$$\boxed{a = 360}$$

choose ① $a + w = 400$

$$360 + w = 400$$

$$-360 \qquad -360$$

$$\boxed{w = 40}$$

The airplane's airspeed is 360 km/hr, and the speed of the wind is 40 km/hr